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A Simple Experimental Analysis on Transportation Problem: A New Approach to Allocate Zero Supply or Demand for All Transportation Algorithm

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ABSTRACT
Basic feasible solution is the initial solution of transportation problem. There are several transportation algorithms to determine feasible solution. These algorithms are only way to get optimal solution because optimal solution obtains from the initial basic feasible solution using some optimality test methods such as MODI-Modified Distribution Method and Stepping Stone Method where MODI is the most efficient method to determine optimal solution. Before applying MODI on feasible solution we have to resolve degeneracy if it occurs and also make sure that allocations (non-negative basic variable) are should not form a closed loop when we draw horizontal and vertical paths from allocated cells to allocated cells. Degeneracy occurs when supply and demand are satisfied simultaneously it can be resolved if one row or column of them is crossed out and another remains with zero supply or demand. Difficulty arise when if all uncrossed out rows or column have (remaining) zero supply or demand. In that case, Vogel's Approximation Method (VAM) and some other methods solve that by allocating these zero supply or demand using Least Cost Method (LCM). But in some problem when we make allocation by LCM in that particular case then closed loop is formed and not possible to apply any optimality test method. So that optimal solution is not possible to determine in that case. In this paper we resolve this particular problem and find a new approach to allocate zero supply or demand so that closed loop will not be formed.
Keywords - Transportation Problem (TP), LCM, VAM, MODI, Degeneracy, Closed Loop.

I. INTRODUCTION

To get the optimal solution of a Transportation Problem first a fall we have to determine basic feasible solution using transportation algorithm. The efficiency of the feasible solution is to determine by the feasible solution is how close to optimal solution. Not only that but also it depends on obtaining non-degeneracy solution. In feasible solution, if number of allocations are not equal to (or less than) $m+n-1$ where m is the number of rows and n is the number of columns, this situation is called "Degeneracy". Degeneracy occurs when supply and demand are satisfied simultaneously which is solved by determining one row or column of them is crossed out and another remaining row or column set up with zero supply or demand. But still if degeneracy occurs then add zero amounts to the lowest unallocated cells. For that above cases the especial situation may be arise when all uncrossed out rows or column have (remaining) zero supply or demand. Vogel's Approximation Method (VAM) solves this by using Least Cost Method (LCM) in that situation. But in some problem, for that particular case, closed loop are formed among allocated cells when this situation solved by LCM. That's why the value of $u_i \; and \; v_j$ called by implied cost $C_{ij} = u_i + v_j$ for all allocated cells are not possible to determine and hence optimal solution can not possible to obtain. In this paper, for that especial case we developed a logical solution and show some numerical example to solve this type of problem using our proposed logical approach for some transportation algorithms.

II. PROPOSED IMPROVEMENT FOR REMAINING ZERO SUPPLY OR DEMAND

For any transportation algorithm, in last allocation supply and demand are must satisfied simultaneously where one row or column is crossed out and another is remains with zero supply or demand. By the rules of transportation algorithm, when exactly one row or column has zero supply or demand then iteration is complete and this zero must not allocate in any cell. That's mean zero supply or demand which comes out from the iteration of last simultaneously satisfied row or column is always out of calculation.

Using this above idea we solve the especial case of transportation problem when all uncrossed out rows or column have (remaining) zero supply or demand which is solved by Least Cost Method (LCM) and formed closed loop so that u_i *and* v_j can not possible to determine for optimal solution by using MODI. And also we proposed an improvement for these remaining zero supply or demand. The improvement is given below:

i. **If all uncrossed out rows or column have (remaining) zero supply or demand then allocate the zero supply or demand by the Least-Cost Method except that zero which comes out from the last simultaneously satisfied row and column.**

Applying this improvement in all transportation algorithms is possible to avoid the forming a closed loop among all allocations and able to determine optimal solution.

III. EXPERIENTIAL ANALYSIS

Consider some special types of Transportation Problems involving the above situation *i.e.* when the last positive allocation has given then there are only some zero supply or demand have remain. In that case we apply our improved logic to allocate the zero supply or demand so that closed loop won't be formed and possible to obtain optimal solution to calculate u_i *and* v_j for MODI.

3.1 EXAMPLE-1:

Consider a Mathematical model of a transportation problem:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	5	3	6	10	**30**
S2	6	8	10	7	**10**
S3	3	1	6	7	**20**
S4	8	2	10	12	**10**
Demand	**20**	**25**	**15**	**10**	

Table-1.1

Solution of Example-1 Using Least Cost Method (LCM):

The iteration when the last positive allocation has done:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	5 **20**	3	6 **10**	10	
S2	6	8	10	7 **10**	
S3	3	1 **20**	6	7	
S4	8	2 **5**	10 **5**	12	
Demand			**0**	**0**	

Table-1.2

Here the cell (S4,D3) is the last simultaneously satisfied positive allocated cell and D3 and D4 are only uncrossed out columns with zero demand. In that case if we allocate zero demand by LCM then cell (S3,D3) is the least cost cell with cost 6 between D3 and D4 and zero demand should be allocate in that (S3,D3) cell and hence one zero demand is remains which must be leave from calculation by the rules of TP algorithm. So the final feasible solution by LCM is given below:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	5 **20**	3	6 **10**	10	
S2	6	8	10	7 **10**	
S3	3	1 **20**	6 **0**	7	
S4	8	2 **5**	10 **5**	12	
Demand				**0**	

Table-1.3

Optimality Test for Example-1 using MODI:

u_i	Sources	Destinations				Supply
		D1	D2	D3	D4	
$u_1 = 0$	**S1**	5 **20**	3	6 **10**	10	**30**
$u_2 = ?$	**S2**	6	8	10	7 **10**	**10**
$u_3 = 3$	**S3**	3	1 **20**	6 **0**	7	**20**
$u_4 = 4$	**S4**	8	2 **5**	10 **5**	12	**10**
	Demand	**20**	**25**	**15**	**10**	
v_j		$v_1 = 5$	$v_2 = -2$	$v_3 = 6$	$v_4 = ?$	

Table:1.4

For this feasible solution of Example-1, we are not able to obtain optimal solution by MODI because the implied costs u_2 *and* v_4 for allocated cells are not possible to calculate from this matrix. So using traditional approach for allocating zero supply or demand can't able to determine optimal solution by MODI.

Solution of Example-1 Using improved logic for allocating zero supply or demand:

If we solve this transportation problem using our proposed improved logic for allocating zero supply or demand by Least Cost Method then it easily resolve.

The iteration when the last positive allocation has done:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	5 **20**	3	6 **10**	10	
S2	6	8	10	7 **10**	
S3	3	1 **20**	6	7	
S4	8	2 **5**	10 **5**	12	
Demand			**0**	**0**	

Table-1.5

Here the cell (S4,D3) is the last simultaneously satisfied positive allocated cell and D3 and D4 are only uncrossed out columns with zero demand. Then by our improved logic that zero supply or demand which comes out from the iteration of last simultaneously satisfied row and column should leave from calculation and remaining zero supply or demand are allocated by LCM. Using this idea it follows that zero demand in D3 column should leave and the remaining zero demand in D4 column allocate this in the lowest cost cell with cost 7 *i.e.* in (S3,D4) cell. The final feasible solution of this TP in given below using our improved idea:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	5 **20**	3	6 **10**	10	
S2	6	8	10	7 **10**	
S3	3	1 **20**	6	7 **0**	
S4	8	2 **5**	10 **5**	12	
Demand			**0**		

Table:1.6

Optimality Test using proposed improvement for Example-1 by MODI:

u_i	Sources	Destinations				Supply
		D1	D2	D3	D4	
$u_1 = 0$	S1	5 **20**	3 **(5)**	6 **10**	10 **(6)**	**30**
$u_2 = 3$	S2	6 **(-2)**	8 **(7)**	10 **(1)**	7 **10**	**10**
$u_3 = 3$	S3	3 **(-5)**	1 **20**	6 **(-3)**	7 **0**	**20**
$u_4 = 4$	S4	8 **(-1)**	2 **5**	10 **5**	12 **(4)**	**10**
	Demand	**20**	**25**	**15**	**10**	
v_j		$v_1 = 5$	$v_2 = -2$	$v_3 = 6$	$v_4 = 4$	

Table:1.7

For this above feasible solution of Example-1, we are able to obtain optimal solution because of all implied costs *i.e.* u_i *and* v_j for allocated cell and opportunity cost for all unallocated cells are possible to calculate. So the improvement for remaining zero supply or demand is successful in that case.

3.1 EXAMPLE-2

Consider another Mathematical Model of a Transportation Problem:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	7	7	8	10	**20**
S2	9	8	10	5	**30**
S3	3	6	1	8	**20**
S4	8	2	10	9	**10**
Demand	**20**	**10**	**20**	**30**	

Table: 2.1

Solution of Example-2 Using Traditional Approach by Vogel's Approximation Method (VAM):

The iteration when the last positive allocation has done:

Sources	Destinations				Supply
	D1	D2	D3	D4	
S1	7 **20**	7	8	**10**	
S2	9	8	10	5 **30**	
S3	3	6	1 **20**	8	
S4	8	2 **10**	10	9	
Demand	**0**	**0**	**0**	**0**	

Table: 2.2

In this solution using VAM the cell (S1,D1) is the last positive allocated cell and four zero demands are remains in D1, D2, D3 and D4 columns. By the rules of transportation algorithm If we allocate zero demand by LCM then first three zero demand should be allocated in the lowest cost cell in (S3,D1), (S3,D2) and (S1,D3) respectively which in shown in Table-2.3. The final table for feasible solution using VAM is given below:

Sources	**Destinations**				**Supply**
	D1	**D2**	**D3**	**D4**	
S1	7 **20**	7	8 **0**	10	
S2	9	8	10	5 **30**	
S3	3 **0**	**6** **0**	**1** **20**	8	
S4	8	2 **10**	10	9	
Demand				**0**	

Table-2.3

Optimality Test for Example-2 using MODI:

u_i	**Sources**	**Destinations**				**Supply**
		D1	**D2**	**D3**	**D4**	
$u_1 = 0$	**S1**	7 **20**	7	8 **0**	10	**20**
$u_2 = ?$	**S2**	9	8	10	5 **30**	**30**
$u_3 = -4$	**S3**	3 **0**	6 **0**	1 **20**	8	**20**
$u_4 = -8$	**S4**	8	2 **10**	10	9	**10**
	Demand	**20**	**10**	**20**	**30**	
v_j		$v_1 = 7$	$v_2 = 10$	$v_3 = 8$	$v_4 = ?$	

Table-2.4

For this feasible solution of Example-2, we are not able to obtain optimal solution by MODI because the implied costs u_2 *and* v_4 for allocated cells are not possible to calculate from this above matrix and $v_3 = 8$ is not verified by v_1 *and* v_2. So using traditional approach for allocating zero supply or demand can't able to determine optimal solution by MODI.

Solution of Example-2 Using improved logic for allocating zero supply or demand for VAM:

If we solve this transportation problem using our proposed improved logic for allocating zero supply or demand for VAM then it can easily resolve. The iteration when the last positive allocation has done:

Sources	**Destinations**				**Supply**
	D1	**D2**	**D3**	**D4**	
S1	7 **20**	7	8 **0**	10	
S2	9	8	10	5 **30**	
S3	3	6 **0**	1 **20**	8 **0**	
S4	8	2 **10**	10	9	
Demand	**0**				

Table: 2.5

Here (S1,D1) cell is the last simultaneously satisfied positive allocated cell and four zero demand are remains in D1, D2, D3 and D4 columns. Then by our improved logic that zero supply or demand which comes out from the iteration of last simultaneously satisfied row and column should leave from calculation and remaining zero supply or demand are allocated by LCM. So that zero demand in D1 column should be leave from this calculation and allocate other zero demands in (S2,D2), (S1,D3) and (S3,D4) respectively by LCM.

Optimality Test using proposed improvement for Example-2 by MODI:

u_i	**Sources**	**Destinations**				**Supply**
		D1	**D2**	**D3**	**D4**	
$u_1 = 0$	**S1**	7 **20**	7 **(-6)**	8 **0**	10 **(-5)**	**20**
$u_2 = -10$	**S2**	9 **(12)**	8 **(5)**	10 **(12)**	5 **30**	**30**
$u_3 = -7$	**S3**	3 **(0)**	6 **0**	1 **20**	8 **0**	**20**
$u_4 = -11$	**S4**	8 **(12)**	2 **10**	10 **(13)**	9 **(4)**	**10**
	Demand	**20**	**10**	**20**	**30**	
v_j		$v_1 = 7$	$v_2 = 13$	$v_3 = 8$	$v_4 = 15$	

Table-2.6

For this above feasible solution of Example-2, we are able to obtain optimal solution because of all implied costs *i.e.* u_i *and* v_j for allocated cell and opportunity cost for all unallocated cells are possible to calculate. So the improvement for remaining zero supply or demand is successful in that case.

IV. CONCLUSION

From the above discussion and analysis we observed that the traditional and existing approach can't calculate the MODI from feasible solution for the especial case when all uncrossed out rows or column have (remaining) zero supply or demand. But if we apply our improved approach that allocating the zero supply or demand by the Least-Cost Method except that zero which comes out from the last simultaneously satisfied row and column then feasible solution does not form a closed loop and possible to obtain optimal solution using MODI.

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